IT 327 – SUPPLEMENTARY MATERIAL CHAPTER 3

AC Electric Circuits

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Chapter 3: AC Electric Circuits

Preview

The DC circuits studied in the previous chapter consist of only two kinds of elements: DC voltage rises (sources) and loads (resistors). While these circuits are very useful, they pale in comparison to the other type of circuits, where varying voltages (AC) are used.

In this chapter, we will study in detail several new terms, most of which were first introduced in Table 1-2 on page 3 of Chapter 1. These include *frequency*, *capacitance*, *inductance*, *reactance*, and *impedance*. We will analyze circuits with unique AC behaviors, and we will use the same laws we have learned before (Ohm's Law, the Power Formula, Kirchoff's Voltage Law, and Kirchoff's Current Law) to analyze them. However, we will need to use complex numbers to perform this analysis, since vector math is essential to correct analysis of AC circuits.

In Chapter 1, we were introduced to the concept of resistance, and using Ohm's Law, we learned to correctly analyze circuits using this concept of resistance. However, resistance is only one of three types of opposition to current flow. The other two we will cover in this chapter are *reactance* and *impedance*; both oppose the flow of current, but they do it in a way that is very different from resistance.

3-1 AC Waveforms and Terms

In reality, AC waveforms can have nearly any shape. But ideally speaking, AC waveforms are of four basic types: sine waves, square waves, triangle waves, and sawtooth waves, as depicted in Figures 3.1 – 3.4.

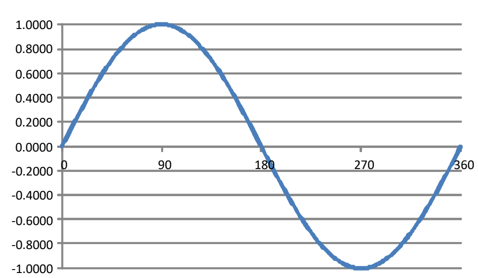


Figure 3-1: An example of one cycle of a sine wave

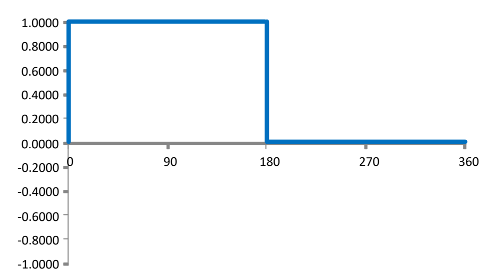


Figure 3-2: An example of one cycle of a square wave

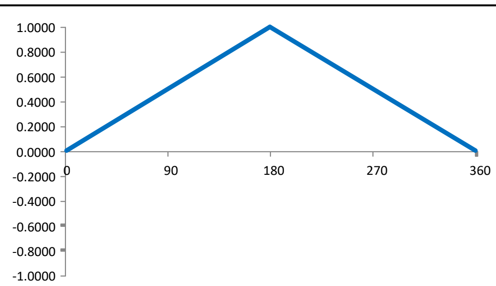


Figure 3-3: An example of one cycle of a triangle wave.

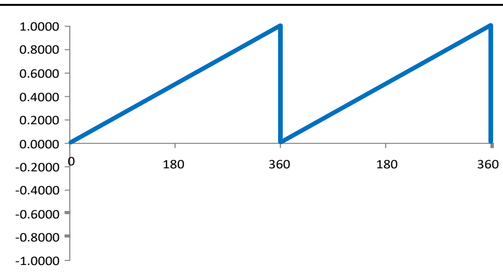


Figure 3-4: An example of two cycles of a sawtooth wave.

A sine wave is the most fundamental type of waveform – all the other types of waveforms can be discussed in terms of sine waves. And since all AC power in the world is sinusoidal, there some terms regarding sine waves with which we should become familiar.

DC voltages have only one way of being specified – Volts(DC). But AC voltages can be specified in three ways: Volts(*peak*), Volts(*peak-peak*), and Volts(*rms*). Figure 3-5 shows the relationships among these three AC voltage specifications. From this figure, it can be seen that $V_{p\text{-}p}$ is the greatest of these three, and that it is exactly twice the value of V_p. V_{rms} is the least of these three, and it is defined as:

$$V_{rms} = 0.7071 \times V_p \qquad \mathbf{3.1}$$

Alternatively, it is also defined as:

$$V_{rms} = \frac{V_p}{\sqrt{2}} \qquad \mathbf{3.2}$$

These definitions are equivalencies.

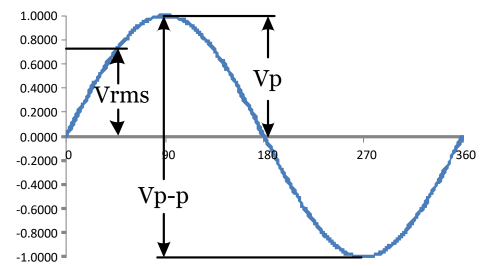


Figure 3-5: The relationships among Volts(peak), Volts(peak-peak), and Volts(rms).

V_{rms} is the root-mean-square (quadratic average) of the sinusoidal voltage, and is the DC equivalent of a sinusoidal voltage. For example, in the USA most AC power is 120 Volts; this is the DC equivalent of the AC voltage, and thus the V_{rms} value. The V_p value for AC power in the USA would then be:

$$V_p = V_{rms} \times \sqrt{2} = 120\,V \times 1.414 = \mathbf{169.68\,V}$$

and the $V_{p\text{-}p}$ value would be:

$V_{p\text{-}p} = 2 \text{ x } V_p = 2 \text{ x } 169.68\text{ V} = \mathbf{339.36\text{ V}}$

As another example, suppose a given AC voltage is specified as 38 $V_{p\text{-}p}$. This would also be equivalent to:

$V_p = V_{p\text{-}p} / 2 = 38\text{ V} / 2 = \mathbf{19\text{ V}}$

and to

$V_{rms} = V_p * 0.7071 = 19\text{ V} * 0.7071 = \mathbf{13.4349\text{ V}}$

An AC sine wave has four specific parameters, one of which is the *amplitude* which we have been calculating. The other parameters are the *period*, the *frequency*, and the *phase*. However, since the period and the frequency have a very simple relationship between them:

freq (f) = 1 / period (t)   and   period = 1 / f

in reality, there are only three parameters to specify a sine wave: amplitude, frequency (or period), and phase.

A sine wave with a frequency of 1.0 MHz would have a period of 1.0 µs (the inverse of 1.0 MHz). Likewise, a sine wave with a period of 1.0 ns would have a frequency of 1.0 GHz (the inverse of 1.0 ns).

Figure 3-6 shows four sine waves, three of which differ from the black sine wave in each of the preceding parameters: period, amplitude, and phase.

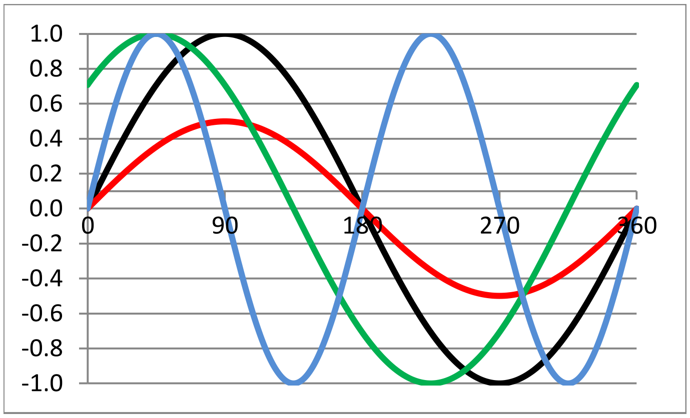


Figure 3-6: Four sine waves, 3 of which differ from the black sine wave in amplitude (red = ½ amplitude of black), phase (green leads black by 45°), and frequency (blue = twice the frequency of black).

3-2 Vectors with AC

With DC, it is easy to add voltages directly. However, Figure 3-6 should give us some indication why adding AC voltages is not so simple. If one does not take into consideration all the parameters of an AC voltage, adding them together would give VERY different (and wrong) results. For example, if we add together the red and black waveforms from Figure 3-6, we get a simple result that the combined waveform has a maximum amplitude of +1.5, and looks like the purple waveform in Figure 3-7. This is an example of a correct addition of two AC waveforms. However, it is only correct because the two waveforms do NOT differ in phase nor in frequency.

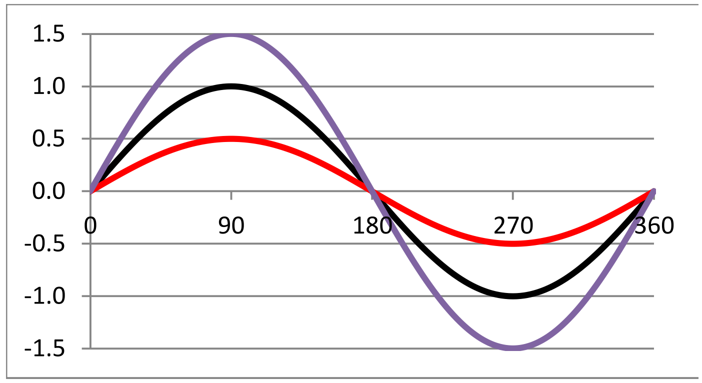


Figure 3-7: Two AC waveforms (the red and the black), correctly summed to equal the purple waveform.

An example of an incorrect addition of two waveforms can be seen in Figure 3-8, where the black and the green waveforms are added to produce the incorrect purple waveform. It is incorrect because it does not take into account the different phase of the green waveform. The correct addition of the black and green waveforms is seen in the orange waveform. Notice how different it is from the incorrect (purple) waveform: it differs both in amplitude and in phase. This is an example of why we need vector math to add together AC waveforms.

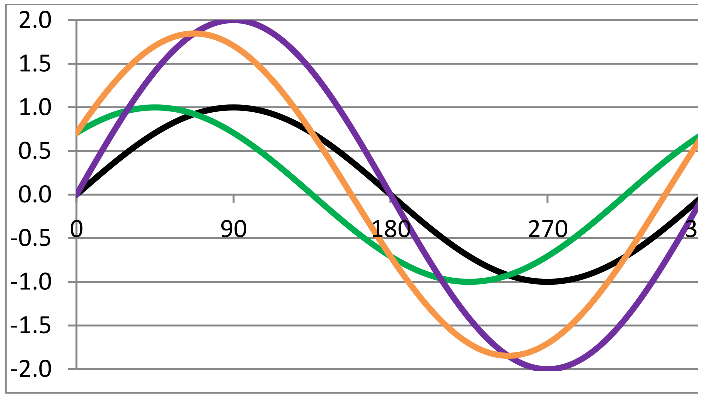


Figure 3-8: Two AC waveforms (the green and the black) incorrectly (purple) and correctly (orange) added together.

Another example of the importance of vectors is found in distances. If one were to ask the question, If you are in Salt Lake City, UT, how far is it fly to Phoenix, AZ? A brief consultation with a map gives the answer of about 500 miles, or 800 km. But there is a very basic built-in assumption in this answer: it assumes you're flying SOUTH. But what is that distance if you're flying NORTH? In that case, the answer is about 24,500 miles, or 39,200 km.

Vectors are simply numbers with BOTH magnitude and direction, or phase. And since AC waveforms possess both magnitude and phase, we must use vectors to correctly work with them mathematically. To express three of the waveforms in Figure 3-6, we would use the following expressions:

V = 1 /0° (meaning 1 V_p, at a phase angle of 0°)

V = 0.5/0° (meaning 0.5 V_p, at a phase angle of 0°)

V = 1/45° (meaning 1.0 Vp, at a phase angle of 45°)

We cannot use this notation for waveforms of different frequencies, but since this is not necessary for the circuits to be analyzed in this text, we will not concern ourselves with this.

Using the above notation, the first part of our vector is the magnitude, and the second part is the phase. This notation is also known as polar notation, and is useful for multiplying and dividing vectors. However, if vectors are to be added or subtracted, it is much more convenient to have them in rectangular notation, as follows:

V = 1 + *j*0 (where the *j* component is the imaginary component on the y axis)

V = 0.5 + *j*0

V = 0.7071 + *j*0.7071

The equations governing conversion from polar to rectangular and vice versa are:

x = z * cos(Θ) $z = \sqrt{x^2 + y^2}$

y = z * sin(Θ) $\Theta = \tan^{-1}(y/x)$

Since these relationship were all anciently derived from right triangles (well over 2,000 years ago), it is helpful to use some triangles to demonstrate these equivalencies. Figure 3-9 shows a right triangle with x = 3 and y = 4. Using the above equations, we can find the polar equivalent:

$z = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \mathbf{5.00}$

$\Theta = \tan^{-1}(4/3) = \mathbf{53.13°}$

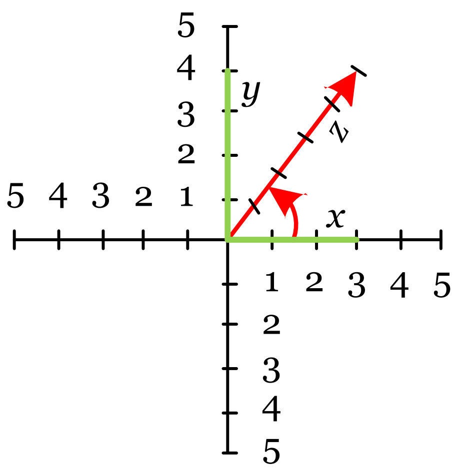


Figure 3-9: Two sides of a right triangle (green), with equivalent magnitude and phase angle (red).

Another right triangle is depicted in Figure 3-10, with z = 10 and Θ = 60°. Using the equations to convert from polar to rectangular form, we find:

x = 10 * cos(60°) = 10 * 0.500 = **5.00**

y = 10 * sin(60°) = 10 * 0.866 = **8.66**

3-3 Vector Math

The previous section introduced why we need to use vectors (complex numbers) to perform mathematical functions with AC. It also introduced how to convert from polar to rectangular notation, and how these complex numbers are written in each of these forms. It is important to remember that polar and rectangular notations are EQUIVALENT – either is correct. For example, in Figure 3-9, we could write this complex number either way:

Rectangular: 3.0 + *j*4.0

Polar: 5.0 /53.13°

Which we use depends on the mathematical operation we need to perform. If we need to add or subtract complex numbers, they should be in rectangular form; if we need to multiply or divide complex numbers, they should be in polar form. And just as a reminder, these operations are performed as follows.

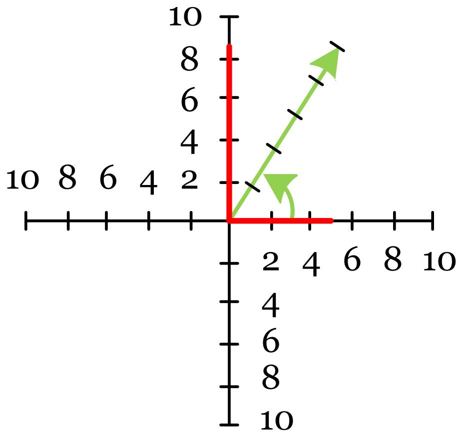


Figure 3-10: The magnitude and phase (green) of the hypotenuse of a right triangle, with the equivalent x and y components (red).

For addition and subtraction, simply line up the x and y portions of the number and perform the addition and subtraction on each portion separately. Two examples follow:

3.00 + *j* 4.00	3.00 + *j*4.00
+ 5.00 + *j* 8.66	- 5.00 + *j*8.66
= 8.00 + *j*12.66	=-2.00 – *j*4.66

For multiplication, multiply the magnitudes and add the angles. For division, divide the magnitudes and subtract the angles. Two examples follow:

5.00 /<u>53.13°</u>	5.00 /<u>53.13°</u>
x 8.66 /60.00°	÷ 8.66 /60.00°
=43.30 /<u>113.13°</u>	= 0.5774 /<u>-6.87°</u>

3-4 Inductors

An inductor is simply a coil of wire, as shown in Figure 3-11. As such, it possesses a concentrated amount of *inductance*. Inductance is the property of electricity in which a <u>change</u> in current is opposed. It is caused by the presence of a magnetic field whenever current is present. Energy is stored in this magnetic field, and changing the magnitude or polarity of this energy requires additional energy, but it is energy which is <u>returned</u> to the electrical circuit.

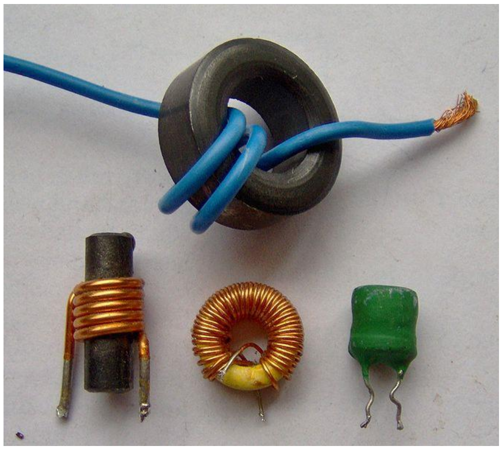


Figure 3-11: An example of four inductors. (from Wikipedia article on *inductors*, June 2012; no author found)

Inductance behaves much like inertia – to change the velocity or direction of a mass requires additional energy, but in a lossless (frictionless) system, that energy is returned when the mass is stopped. The same is true of current in an inductor – to change the magnitude or polarity of the current requires energy, but that energy is stored in the magnetic field associated with the current, and is thus returned to the circuit when the current is stopped. Thus, inductance opposes changes in the current, but does not impose losses on the circuit. This also means that inductors do <u>not</u> oppose DC, but they <u>do</u> oppose AC. The amount of inductance is measured in Henries (after Joseph Henry, an American scientist who discovered electromagnetic induction).

The form of opposition provided by inductors is known as inductive *reactance*, and its units are Ohms (Ω). The amount of reactance of an inductor is a function of the frequency and the inductance, according to the formula:

$$X_L = 2\pi f L \qquad \textit{Inductive Reactance} \qquad 3.3$$

For example, an inductor of 45 µH would give the following reactance at 2.5 MHz:

X_L = 2 * π * 2.5 MHz * 45 µH = **706.86 Ω**

At a frequency of 25 MHz, this same inductor would give a reactance of:

X_L = 2 * π * 25 MHz * 45 µH = **7.0686 kΩ**

At 10x higher frequency, the inductor gives 10x more reactance. Likewise, at 1/10 the frequency, the inductance would be 1/10:

X_L = 2 * π * 0.25 MHz * 45 µH = **70.686 Ω**

Inductors also cause the current and the voltage to shift out of phase, by 90°. This is because, while an inductor opposes changes in the current, it does <u>not</u> oppose changes in voltage. So, the voltage undergoes NO phase shift, while the current is delayed by 90°. Thus, in inductive circuits, the voltage leads the current. This is commonly remembered with the acronym ELI, indicating that E (voltage) leads I (current) in inductive (L) circuits.

One frequent question asked about inductors is, What are they used for? One of the most common applications of inductors is to keep AC signals from interfering with other circuits (particularly

DC circuits), since inductors oppose AC, but do not oppose DC. Another major application of inductors is in frequency-selective circuits, such as those that make possible all modern communication devices and systems.

3-5 Series RL Circuits

With the preceding brief introduction to inductors, together with our refresher on complex numbers, we are now prepared to do an analysis on a simple RL series circuit. The circuit of Figure 3.12 will suffice for this example.

We will perform the same kind of analysis we did in Chapter 1, wherein we find the total opposition, the total current, all the voltage drops and powers. However, first we need to introduce a new concept, which is *impedance*. Impedance is the total opposition that a circuit possesses, consisting of both resistance and reactance. However, since resistance does NOT cause phase shift while reactance DOES, they must be added as vectors.

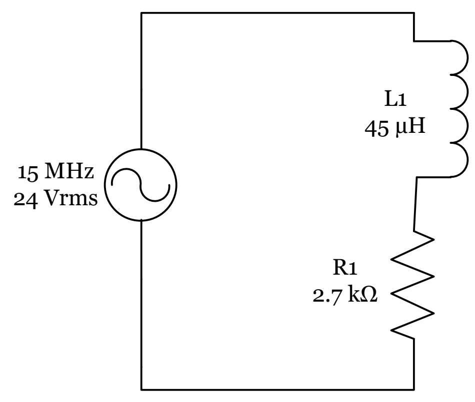


Figure 3-12: A simple series RL circuit to be analyzed.

First, we must find the reactance of the inductor, as we are only given its inductance, and its reactance is frequency-dependent. This if found using Equation 3.3:

$X_L = 2\pi fL = 2 * \pi * 15\text{ MHz} * 45\ \mu\text{H} =$ **4.241 kΩ**

Next we find the impedance of the circuit. Written in rectangular form, this is simply:

$Z_T =$ **2.70 kΩ +*j*4.241 kΩ**

In polar form, this same impedance is:

$Z_T =$ **5.028 kΩ/57.52°**

To find the total current, we must use Ohm's Law as before, but instead of resistance, we must use impedance, since our circuit consists of both resistance and reactance:

$I_T = E_T / Z_T = 24\ V_{rms}/\underline{0°} / 5.028\ k\Omega/\underline{57.52°} =$ **4.773 mA/-57.52°**

In the above equation, we used $E_T = 24\ V_{rms}/\underline{0°}$; but the phase angle of the applied voltage (V_T) was not given in Figure 3-12. The reason we knew that the phase angle of the applied voltage was 0° is because the applied voltage is always our reference voltage – all other phase angles are measured with reference to it – so in this text it is always 0°.

Now that we know the current in the circuit, we can calculate the voltage drops using Ohm's Law, as long as we remember that inductive reactance is always at +90° with respect to the applied voltage, since it causes the voltage to lead the current.

$V_{L1} = I_{L1} * X_{L1} = 4.773\text{ mA}/\underline{-57.52°} * 4.241\ k\Omega/\underline{+90°} =$ **20.243 V/32.48°**

$V_{R1} = I_{R1} * R_1 = 4.773\text{ mA}/\underline{-57.52°} * 2.70\ k\Omega/\underline{0°} =$ **12.89 V/-57.52°**

Since we always like to check our answers, we check to see if the sum of the voltage drops ($V_{L1} + V_{R1}$) is equal to the applied voltage (Kirchoff's Voltage Law). At first glance, it might appear that something is wrong, since it looks like these voltage drops sum to >32 V, yet only 24 V is applied. However, we must remember to add the voltage drops as vectors; when we do this, everything comes out OK. But the voltage drops we have calculated are in polar form, and to add, we need them in rectangular form, so we must first convert them:

$V_{L1} = 17.077\text{ V} + j10.871\text{ V}$ $V_{R1} = 6.922\text{ V} - j10.874\text{ V}$

$V_T = V_{L1} + V_{R1} = (17.077\text{ V} + j10.871\text{ V}) + (6.922\text{ V} - j10.874\text{ V}) =$ **23.999 V – *j*0.003 V**

Throughout this analysis, we have rounded numbers many times. So, if we ignore these rounding errors, we see that the above V_T is indeed equal to the 24V/0° of the applied voltage:

23.999 V – *j*0.003 V ≈ 24 V – *j*0V = **24 V/0°**

Figure 3-13 gives an example of the vectors being dealt with in this analysis.

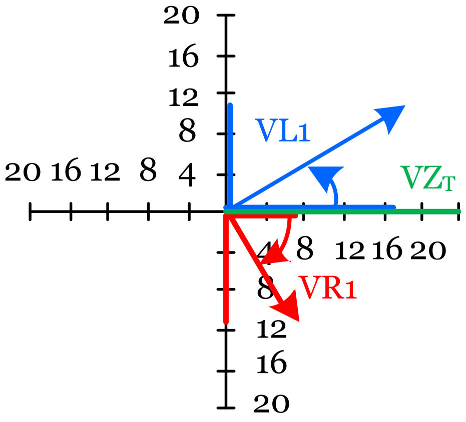


Figure 3-13: The vector voltages being added to test against Kirchoff's Voltage Law. The vertical blue and red vectors cancel each other out when added, and adding the horizontal blue and red vectors results in the green vector, which is the total voltage.

Next we must consider the power in this circuit. Figure 3-14 shows the phase relationships between the applied voltage, the resulting current lagging by 90°, and the power in an inductor. There is something very unique to note here, and that is that the power (the green line) is both positive and negative. What does negative power mean? It means the inductor is giving power back to the circuit. And note that the amount of positive power is equal to the amount of negative power – the net power used by an inductor is zero!

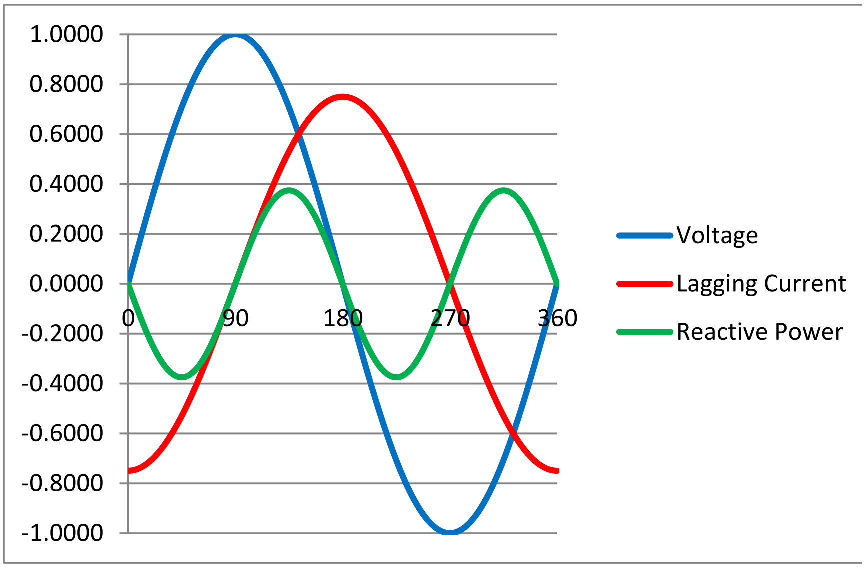
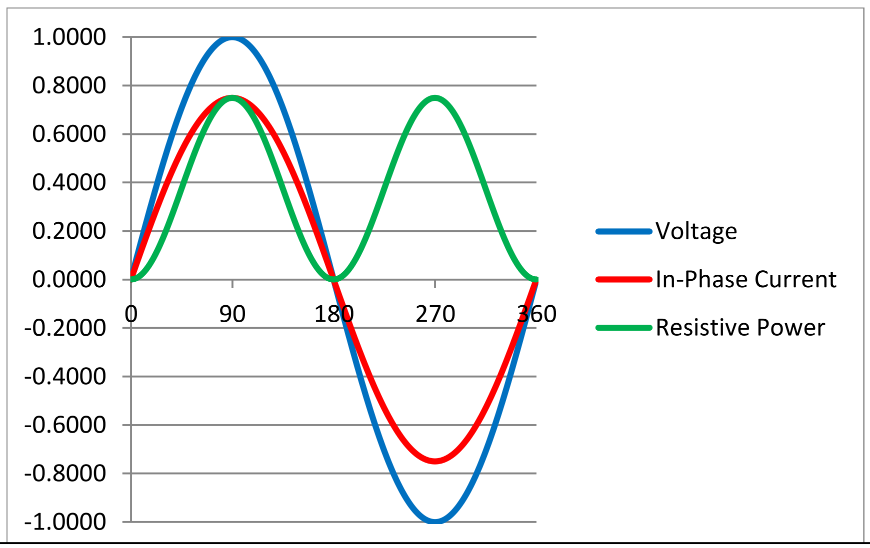


Figure 3-14: The phase relationships between the applied voltage, the resulting current, and the power in an inductor

Contrast this with Figure 3-15, which shows the phase relationships between the applied voltage, the resulting in-phase current, and the power in a resistor. Note that the power is always positive, meaning that



	Opposition		Voltage		Current	
	Polar	Rectangular	Polar	Rectangular	Polar	Rectangular
Total	**5.028 kΩ/57.52°**	**2.70 kΩ +*j*4.241 kΩ**	24 V/0°	24 V + *j*0V	**4.773 mA/-57.52°**	**2.563 mA – *j*4.026 mA**
L1	**4.241 kΩ/90°**	**0 Ω + *j*4.241 kΩ**	**20.243 V/32.48°**	**17.07 V + *j*10.87 V**	**4.773 mA/-57.52°**	**2.563 mA – *j*4.026 mA**
R2	2.7 kΩ/0°	2.7 kΩ + *j*0 Ω	**12.89 V/-57.52°**	**6.922 V – *j*10.87 V**	**4.773 mA/-57.52°**	**2.563 mA – *j*4.026 mA**

Table 3-1: All the values for the circuit of Figure 3-12. The bold values are the values calculated; normal **values were given.**

the resistor is always using power, and never gives any of it back. This is a characteristic difference between a reactive circuit and a resistive circuit.

The net result of this look at power in reactive circuits is that we do not need to calculate the power in an inductor – it is always zero. And power in reactive circuits, in general, is a topic that is outside the scope of this text, because we would have to get into several concepts that are simply not necessary for the purposes of this introduction.

So the conclusion of the analysis of the circuit in Figure 3-12 are the values shown in Table 3-1, which includes the rectangular and polar forms of each value.

3-6 Capacitors

In its most basic form, a capacitor is simply two conductors, separated by an insulator (usually referred to as the *dielectric* (a term which means it opposes electricity). Capacitors store energy in the form of an *electrostatic field*, which is simply a voltage field. Capacitors come in many sizes and types; an assortment of them is shown in Figure 3-16.

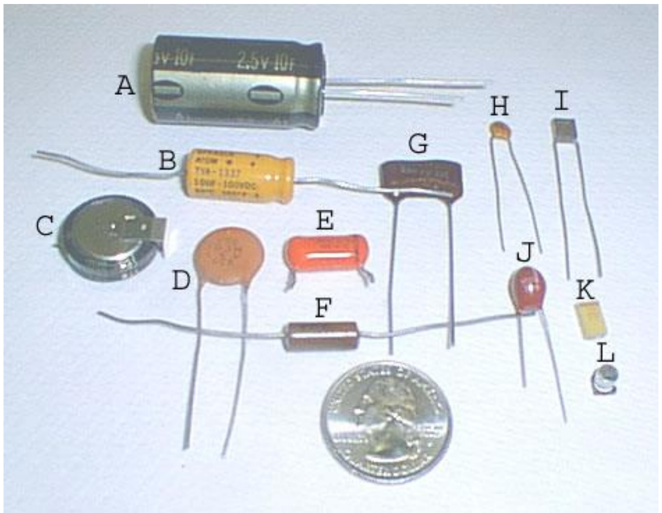


Figure 3-16: An assortment of various types and sizes of capacitors. A=Aluminum electrolytic supercapacitor; B=Aluminum electrolytic; C= Aluminum electrolytic supercapacitor; D=Mica; E=Plastic; F=Plastic; G=Ceramic; H=Mica; I=Ceramic; J=Tantalum electrolytic; K=Ceramic; L=Aluminum electrolytic.

A capacitor behaves much like a two-ported air reservoir, as depicted in Figure 3-17. If you push air into the left side, the diaphragm will displace to the right, as shown in Figure 3-18. This process of pushing air into the reservoir takes a bit of time, and in the process displaces all the air previously in the right side of the reservoir.

If we cap both ends of the reservoir, the result is that we have stored a small amount of pressurized air in it. If we remove the caps, a small puff of air would exit from the left side, and the diaphragm would return to its normal position as depicted in Figure 3-17.

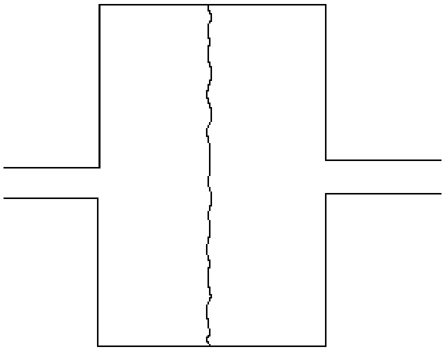


Figure 3-17: A drawing of a 2-ported air reservoir, with a stretchable and air-tight diaphragm in the middle.

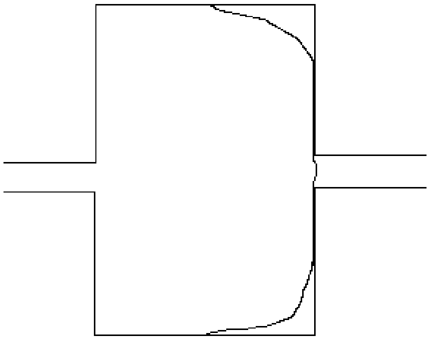


Figure 3-18: The 2-ported air reservoir with the diaphragm stretched and displaced to the right.

So, while air cannot flow continuously through this reservoir, it does behave as though there is air going through it while it is being filled, since the air on the right side is displaced and exits on the right. But once the reservoir is filled, no more air exits on the right.

This is the behavior of capacitors. They behave as though electricity is flowing through them during the time we are charging them, but once they are fully charged, no electricity flows through them. And if we were to rapidly change the voltage being applied to charge the capacitor, it would behave as if the electricity were flowing in both directions with little opposition. Thus, capacitors oppose changes in voltage, while they do not oppose changes in current; this is exactly opposite to inductors. This also means that in a capacitor, the current leads the voltage, which is remembered with the mnemonic ICE (current [I] leads the voltage [E] in Capacitive circuits). The amount of voltage lag in capacitive circuits is -90°.

This also means that capacitors oppose DC, while they oppose AC much less. Their opposition, as in inductors, is called reactance; the units are Ohms (Ω). The equation to compute their reactance is:

$$X_C = \frac{1}{2\pi f C} \qquad \textbf{\textit{Capacitive Reactance}} \qquad \mathbf{3.4}$$

This equation looks very similar to Equation 3.3; they both are equations to compute reactance; they both include 2, π, and frequency. But it is different in that the capacitive reactance is **inversely** proportional to frequency, while inductive reactance is **directly** proportional to frequency.

The amount of capacitance is measured in Farads, named after Michael Faraday, an early experimenter in electromagnetism. One Farad is specified to be the amount of capacitance necessary to store one Coulomb of electrons (1 Coulomb = 6.241×10^{18} electrons) at an applied voltage of 1 Volt. Typical capacitors range from 100 pF to 100 µF, although in supercapacitors, they range up to about 1000 F.

Capacitors, like inductors, are used to separate DC from AC, and are heavily used in frequency-selective circuits. Because they can supply a large amount of current very quickly (whereas batteries cannot), they are also used to store small amounts of charge, as for the flash in cameras.

We should take a few minutes and go over some examples of Equation 3.4. If a 250 pF capacitor is operated at 10 MHz, what would its reactance be?

X_C = 1 / (2 * π * 10 MHz * 250 pF) = **63.66 Ω**

If this same capacitor were operated at 100 MHz, its reactance would go DOWN to:

X_C = 1 / (2 * π * 100 MHz * 250 pF) = **6.366 Ω**

But if we operate this capacitor at 1.0 MHz, its reactance goes UP to:

X_C = 1 / (2 * π * 1.0 MHz * 250 pF) = **636.6 Ω**

3-7 Series RC Circuits

With the preceding brief introduction to capacitors, we are now prepared to do an analysis on a simple RC series circuit. The circuit of Figure 3-19 will suffice for this example.

We will perform the same kind of analysis we did for inductors, wherein we find the total opposition, the total current, all the voltage drops and powers.

First, we must find the reactance of the capacitor, as we are only given its capacitance, and its reactance is frequency-dependent. This is found using Equation 3.4:

X_C = 1/2πfC = 1 / (2*π*18 kHz * 27 nF = **327.5 Ω**

Next we find the impedance of the circuit. Written in rectangular form, this is simply:

Z_T = **430 Ω – *j*327.5 Ω**

In polar form, this would be:

Z_T = **540.5 Ω/-37.29°**

Next we calculate the current, using Ohm's Law:

$I_T = E_T / Z_T$ = 24 V_{rms}/0° / 540.5 Ω/-37.29° =

44.40 mA/37.29°

The voltage drops across the capacitor and resistor are calculated as before in the inductors section:

$V_{C1} = I_{C1} * X_{C1}$ = 44.40 mA/37.29° * 327.5 Ω/-90° =

14.54 V/-52.71°

$V_{R1} = I_{R1} * R_1$ = 44.40 mA/37.29° * 430 Ω/0° =

19.09 V/37.29°

In rectangular form, these voltages are:

V_{C1} = 8.809 V – *j*11.57 V V_{R1} = 15.19 V + *j*11.57 V

Adding these together as shown in Figure 3-20 shows that the sum of the voltage drops does equal the applied voltage, as required by Kirchoff's Voltage Law.

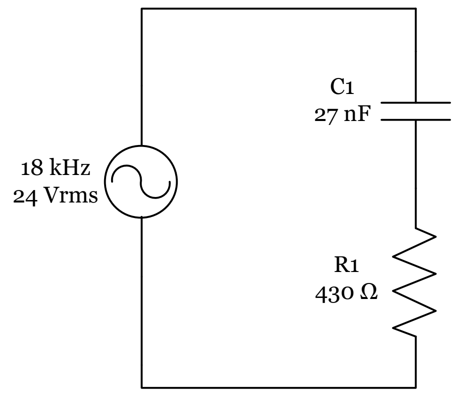


Figure 3-19: A simple series RC circuit for analysis.

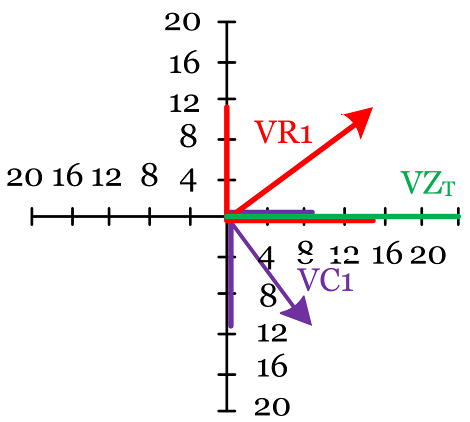


Figure 3-20: A visualization of the voltage vectors being summed to test against Kirchoff's Voltage Law.

3-8 Series RLC Circuits

Series RL and RC circuits are used heavily in separating DC from AC, and as filters (one type of frequency-selective circuit). A final type of frequency-selective circuit is the series RLC circuit, which is dominated by its use in resonant frequency applications – which will be covered in the next section.

In this section we will do an analysis of a simple series RLC circuit: Figure 3-21. We should acknowledge that parallel RL, RC, and RLC circuits are also used, but will not be analyzed in this text.

The first step in this analysis is to calculate the reactance of both reactive elements, C_1 and L_1.

C_1 = 1 / (2 * π * 455 kHz * 200 pF) = **1.749 kΩ**

L_1 = 2 * π * 455 kHz * 245 μH) = **700.4 Ω**

Summing all the opposition in this circuit gives us Z_T:

0.0 Ω – *j*1.749 kΩ
0.0 Ω + *j*700.4 Ω
+ 430 Ω + *j*0.0 Ω

430 Ω – *j*1.049 kΩ , or **1.134 kΩ/-67.71°** in polar form.

We then use this to calculate the total current:

I_T = 15 V/0° / 1.134 kΩ/-67.71° = **13.23 mA/67.71°**

Using this current, we then calculate the voltage drops for each element of the circuit:

$V_{C1} = I_{C1} * X_{C1} = 13.23\text{ mA}\underline{/67.71°} * 1.749\text{ k}\Omega\underline{/-90°} = \mathbf{23.13\text{ V}\underline{/-22.29°}}$ (= 21.41 V – j8.775 V)

$V_{L1} = I_{L1} * X_{L1} = 13.23\text{ mA}\underline{/67.71°} * 700.4\ \Omega\underline{/+90°} = \mathbf{9.265\text{ V}\underline{/157.7°}}$ (= -8.573 V + j3.514 V)

$V_{R1} = I_{R1} * R_1 = 13.23\text{ mA}\underline{/67.71°} * 430\ \Omega\underline{/0°} = \mathbf{5.688\text{ V}\underline{/67.71°}}$ $\underline{(= 2.157\text{ V} + j5.263\text{ V})}$

Summing the three voltage drops gives: **14.99 V + *j*0.002 V**

Ignoring rounding errors, this is the same as the applied voltage of 15.00 V +*j*0.00 V.

Figure 3-22 shows the voltages being summed and their resultant equivalent to the applied voltage, as required by Kirchoff's Voltage Law. A few things should be noted in this diagram. First, note that although the blue, red and purple phasors are rotated clockwise from the x-y ordinates, still the blue phasor (V_{L1}) is 180° opposite the purple phasor (V_{C1}), and the red phasor (V_{R1}) is perpendicular to both of them. This shows the correctness of the solution, for this phase relationship MUST be true in RLC circuits.

Another thing to note in this diagram is that the voltage drops on the x-axis do indeed sum to 15 Volts, and the voltage drops on the y-axis do indeed sum to 0 Volts, again as required in such RLC circuits.

One final thing to note is that in RLC circuits, at certain frequencies the individual voltage drops can exceed the applied voltage. We see this in VC1, which at 23.13 Volts is significantly greater than the 15 Volts applied. We will see this even more in resonant circuits, in the next section.

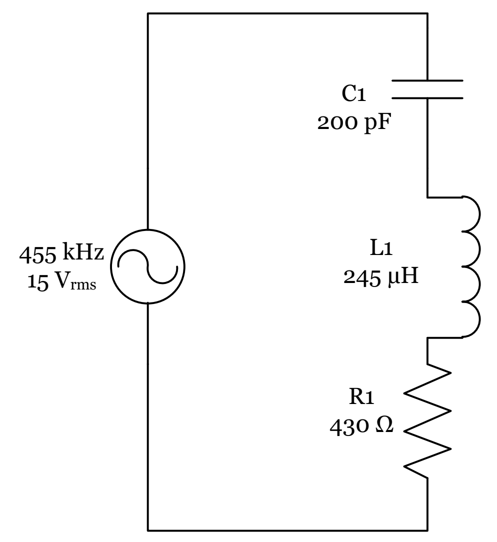


Figure 3-21: A simple RLC circuit for analysis.

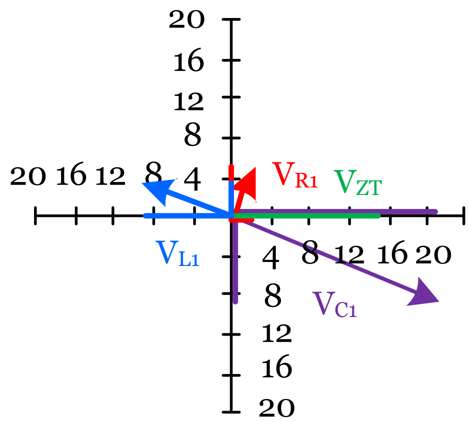


Figure 3-22: A visualization of the voltage vectors being summed to test against Kirchoff's Voltage Law.

3-9 Resonance and Frequency-Selective Circuits

In general, resonance is a characteristic in which there is minimal opposition. We see resonance sometimes in mechanical systems, most notably in the collapse of the Tacoma Narrows Bridge in November 1940 (just visit YouTube and look for this title). In electric circuits, it occurs at the frequency where the inductive reactance equals the capacitive reactance, and since they are 180° opposite each other, at this frequency they cancel each other out.

The frequency at which this occurs can be found simply by setting $X_L = X_C$, and then solving for f (frequency):

$$X_L = 2\pi fL = 1/2\pi fC = X_C$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \qquad \textbf{\textit{Resonant Frequency}} \qquad \textbf{3.5}$$

Note that the resonant frequency depends only on the value of the inductance and the capacitance. For example, in the circuit of Figure 3-21, we would have a resonant frequency of:

f_r = 1 / (2 * π * [√245 µH * 200 pF]) = **719.0 kHz**

Just to verify, let's calculate the respective reactances at this frequency:

X_L = 2 * π * 719.0 kHz * 245 µH = **1.107 kΩ**

X_C = 1 / (2 * π * 719.0 kHz * 200 pF) = **1.107 kΩ**

And under these conditions, the only limit to the current is the resistance, giving:

I_T = 15 V / 430 Ω = **34.88 mA**

which is much greater than the 13.23 mA in this same circuit at 455 kHz. This rise in current is characteristic of series resonant circuits, and is highly desirable in frequency-selective circuits, such as those in cell phones, radios, television, WiFi, etc.

There is also a resonant voltage rise. For example, at this resonant frequency with I = 34.88 mA and reactances of 1.107 kΩ for the inductor and the capacitor, each of these elements now drops:

V_{L1} = V_{C1} = 34.88 mA * 1.107 kΩ = **38.62 V**

which is more than twice the applied voltage!

Resonance is so desirable in frequency-selective circuits that much effort is devoted to making these circuits have more favorable characteristics. The most favorable characteristic of such a circuit is that it respond only to a single frequency, or at most a narrow range of frequencies. This selectivity is primarily determined by the resistance of the inductor, and is a function of its *parasitic* resistance. This deserves further explanation.

All electronic components have a specific desired property; resistors have resistance, inductors have inductance, and capacitors have capacitance. But it is also true that resistors have inductance and capacitance; that inductors have resistance and capacitance; and that capacitors have resistance and inductance. These extra properties are <u>not</u> desirable, but are there simply because it is impossible to make an electronic component without all three of these properties. Because they are not desirable but are present, they are termed parasitic. In general, resistors have very little parasitic inductance and capacitance, and capacitors have very little parasitic resistance and inductance. And inductors have relatively little capacitance, but they DO have a significant amount of resistance. It is this resistance in inductors that limits the selectivity of resonant circuits.

In inductors, this parasitic resistance is significant enough that inductors are rated on it. The *quality factor* of an inductor (the symbol is Q) is simply the ratio of the reactance of the inductor compared to its resistance:

$$Q = \frac{X_L}{R_L} \qquad \textbf{\textit{Quality Factor (Q)}} \qquad \textbf{3.6}$$

which also means it is a function of frequency (since X_L is a function of frequency). For example, in the circuit of Figure 3-21, if R1 were the resistance of the inductor, we would have a Q of:

Q = 700.4 Ω / 430 Ω = 1.63

This is very low, as an ideal inductor would have no resistance, and thus a Q of ∞. In practical circuits, a Q of 10 or more is generally considered good, while a Q of less than 10 is generally considered a low Q.

In the circuit of Figure 3-21, if we could lower the resistance of the inductor to 50 Ω, and raise the resonant frequency to 25 MHz, we would have a greatly improved Q:

Q = (2 * π * 25 MHz * 245 μH) / 50 Ω = 38.48 kΩ / 50 Ω = **769.7**

In summary, there are three things that characterize the resonant frequency in electronic circuits:

1. $X_L = X_C$
2. $Z_T = R$ (because the reactances cancel each other out)
3. Phase shift = 0° (because there is no net reactance)

3-10 Transformers

A coil of wire with current passing through it will produce an electromagnetic field, as discussed in section 3-4 on inductors. If this current is changing with time (AC), it will induce a voltage in a nearby wire (see Figure 3-23), according to Faraday's law of electromagnetic induction:

$$EMF = -N\frac{d\Phi}{dt} \qquad \textit{Faraday's law of electromagnetic induction} \qquad 3.7$$

Remembering that EMF = electromotive force (also known as voltage), and that Φ = magnetic field strength, we see that the nearby wire will have a voltage induced in it ONLY if the magnetic field is changing in time (if $d\Phi/dt = 0$, EMF = 0). In other words, if we pass a DC current through the coil (*instead* of AC), there will be NO voltage induced on the nearby wire, because $d\Phi/dt = 0$.

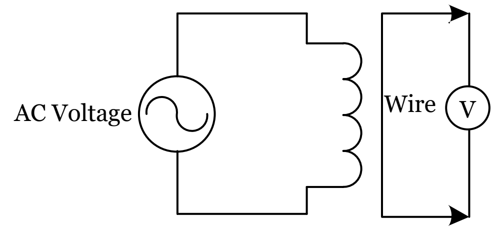


Figure 3-23: A coil of wire, stimulated by an AC voltage and thus producing an AC current, will induce a voltage in a nearby wire.

In Faraday's law of electromagnetic induction, N = the number of coils (or *turns*) of wire. Thus we see that one way to increase the induced voltage is to coil the nearby wire into several loops, each loop increasing the induced voltage. This is exactly how transformers work.

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \qquad \textit{Transformer voltage equation} \qquad 3.8$$

In a transformer, the driven coil is called the *primary* side of the transformer, and the load coil (to which we attach the load) is called the *secondary* side of the transformer. So, according to the Transformer equation, if we have 100 turns on the driven coil (the coil on the left, or N_p) and 100 turns on the load coil (the coil on the right, or N_s), the voltage measured on the right will equal the voltage on the left.

If the secondary of the transformer has 10 turns while the primary has 100 turns, $N_s/N_p = 1/10$, which means the voltage on the secondary would be 1/10 the voltage on the primary. This is known as a *step-down* transformer, as the voltage is stepped down.

If the secondary of the transformer has 100 turns while the primary has 10 turns, $N_s/N_p = 10$, which means the voltage on the secondary would be 10 times the voltage on the primary. This is known as a *step-up* transformer, as the voltage is stepped up. These two actions of stepping voltage up or down are two of the four functions of transformers.

But our intuition tells us something is incomplete in this last example of a step-up transformer. For example, we see that such a transformer would give us 100 V on the secondary with only 10 V on the primary – but that seems like free energy! Can we really get more energy out of a transformer than we put in? Our intuition is correct – we cannot get more energy out than we put in. So something else must be happening. In reality, an *ideal* transformer gives the same power out as the power put in, meaning the transformer has no losses. So, if the power out is equal to the power in, yet the voltage out is 10 times the voltage in, and since power is the product of current and voltage (P = I*E), this means that the current must be going down by the same amount that the voltage goes up. This gives us the complete transformer equation, as given in equation 3.9.

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} \qquad \textit{Complete Transformer equation} \qquad 3.9$$

For example, let's say that a given transformer has a *turns ratio* (N_s/N_p) = 20, and that N_s = 100. Solving for N_p, we get:

$100/ N_p = 20$; $N_p = 100/20 =$ **5**,

which makes this a step-up transformer.

So if we use this transformer with V_p = 5 V and I_p (ampacity) = 10 A, we would get:

$V_s = 20\ V_p = 20 * 5 = 100$ V

But the current (ampacity) goes down:

$I_s = I_p/20 = 10$ A$/20 = 0.5$ A

Step-up and step-down transformers are heavily used throughout the world for delivering AC electricity. This is because the losses in a transmission wire are:

$P = I^2R$

To reduce these very substantial losses, we can step the current down while stepping the voltage up. For example, if we step up the voltage 10x, we step the current down 10x; but this reduces the power losses in the wire by I^2, which means we get a 100x reduction in these losses. Thus, the general rule of power generation is to keep the voltage as high as practical for as long as practical, stepping the voltage down as you get closer to where the power will be used. Figure 3-24 has an example schematic of how power is delivered to room 140 CTB from the generating station near Huntington, UT. In this diagram, we see

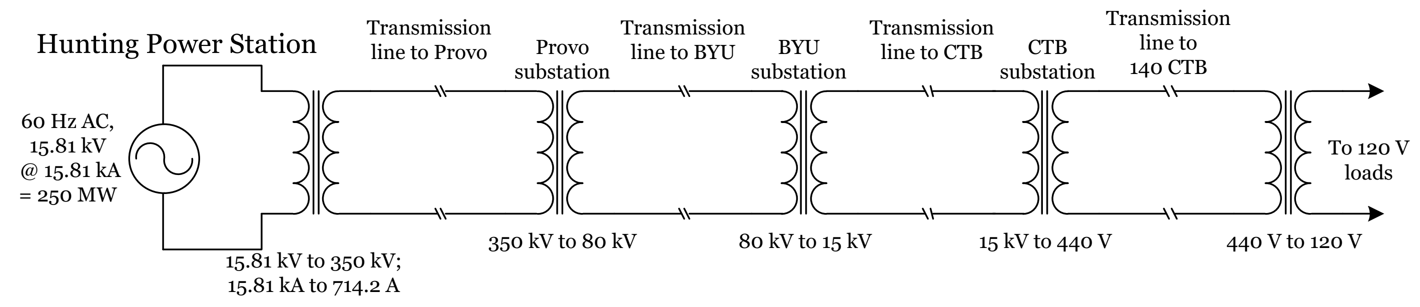


Figure 3-24: Schematic diagram of how power is delivered from the Huntington power station to room 140 CTB. The numbers given are approximate.

one step-up transformer (the one at the Hunting power station), and four step-down transformers (all the others), demonstrating this idea of keeping the voltage as high as practical for as long as practical to reduce the I^2R losses.

The other two uses for transformers are isolation and impedance matching. Isolation transformers are transformers with a turns ratio of 1, meaning that the voltage and current are not changed at all. So if the voltage and current are not changed, what good is such a transformer? This is a bit of a specialty application, but if you have an AC signal which is important (it contains information), but you don't want DC to be transmitted with the AC, you can put such a signal through an isolation transformer. The AC will go through the isolation transformer unchanged, but the DC cannot go through a transformer.

Impedance matching is also a bit of a specialty application, but is very common in high-frequency communication. The equation governing this matching effect is:

$$\frac{N_s}{N_p} = \sqrt{\frac{Z_L}{Z_S}} \qquad \textit{Transformer Impedance Matching} \qquad 3.10$$

Thus, given the number of turns on the primary and the impedances to be matched, the number of turns on the secondary can be found:

$$N_s = \sqrt{\frac{Z_L}{Z_S}} \times N_P$$

Summary on Transformers

1. Transformers do not work with DC, but only with AC.
2. Transformers work by electromagnetic induction (Faraday's law).
3. Transformers are used for four applications:
   a. Stepping voltage up and current down
   b. Stepping voltage down and current up
   c. Isolating DC from AC
   d. Impedance matching

Problems

1. What is the period of a sine wave whose frequency is 4.5 MHz? (5 points)
2. What is the frequency of a square wave whose period is 7.8 ns? (5 points)
3. If a sine wave has an amplitude of 6.5V_{rms}, what is its amplitude in V_p and V_{p-p}? (10 points)
4. If a voltage vector is 14.8V/$\underline{30°}$, what is that vector in rectangular form? (5 points)
5. If an impedance is 1.456 kΩ +*j*4.312 kΩ , what is that impedance in polar form? (5 points)
6. What is reactance? Be sure to include both capacitive and inductive reactance in your definition. (5 points)
7. Describe how an inductor stores energy. (5 points)
8. What is the reactance of a 230 µH inductor at 85 MHz? (5 points)
9. A simple series RL circuit (as in Figure 3-12) has L1 = 45 nH, R1 = 25.3Ω, f = 506 MHz, and V = 15.0 V_{rms}. Find the impedance and current in this circuit. (10 points)
10. Describe how a capacitor stores energy. (5 points)
11. What is the reactance of a 3.50 nF capacitor at 2.65 MHz? (5 points)
12. A simple series RC circuit (as in Figure 3-19) has C1 = 30 pF, R1 = 255 Ω, f = 45 MHz, and V = 20.0 V_{rms}. Find the impedance and current in this circuit. (10 points)
13. A simple series RLC circuit (as in Figure 3-21) has C1 = 15 nF, R1 = 4.6 Ω, L1 = 2.4 µH, f = 500 kHz, and V = 17.0 V_{rms}. Find the impedance, current, and voltage drops for each element of this circuit. (25 points)
14. What is the resonant frequency for the circuit in Problem #13? (5 points)
15. What is resonance in series RLC circuits? (5 points)
16. What is the Q of a 850 nH inductor at 350 MHz, if the resistance of the inductor is 75 Ω? (5 points)
17. A given step-down transformer has a turns ratio of 15, an output voltage of 20 V_{rms}, and an output ampacity of 15 A_{rms}. What is the input voltage and ampacity for this transformer? (10 points)

Answers to Chapter 3 Numerical Problems

1. 222.2 ns
2. 128.2 MHz
3. 9.191 V_p; 18.38 V_{p-p}
4. 12.82 V +*j*7.4 V
5. 4.551 kΩ/$\underline{71.34°}$
8. 122.8 kΩ
9. Z=25.3 Ω +*j*143.1Ω = 145.3 Ω/$\underline{79.97°}$;
   I = 103.2 mA/$\underline{-79.97°}$ = 17.98 mA – *j*101.7 mA
11. 17.16 Ω
12. Z = 255 Ω – *j*117.9 Ω = 280.9 Ω/$\underline{-24.81°}$;
    I = 71.19 mA/$\underline{24.81°}$ = 64.62 mA + *j*29.87 mA
13. Z_T = 4.6 Ω –*j*13.68 Ω, = 14.43 Ω/$\underline{-71.41°}$
    I_T = 1.1781 A/$\underline{71.41°}$
    V_{R1} = 5.419 V/$\underline{71.41°}$
    V_{L1} = 8.883 V/$\underline{161.41°}$
    V_{C1} = 25 V/$\underline{-18.59°}$
14. f_r = 838.82 kHz

16. Q = 24.923 (no units)
17. V_p = 300 V_{rms}
    I_p = 1.0 A_{rms}